In the single-line, multiserver, single-phase model, customers form a single line and are served by the first server available. The model assumes that there are s identical servers, the service time distribution for *each server* is exponential, and the mean service time is $1/\mu$. Using these assumptions, we can describe the operating characteristics with the following formulas:

$$s = \text{the number of servers in the system}$$

$$p = \frac{\lambda}{s\mu} = \text{the average utilization of the system}$$

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!}\left(\frac{1}{1-p}\right)\right]^{-1} = \text{the probability that no customers are in the system}$$

$$L_Q = \frac{P_o(\lambda/\mu)^s p}{s!(1-p)^2} = \text{the average number of customers waiting in line}$$

$$W_Q = \frac{L_Q}{\lambda} = \text{the average time spent waiting in line}$$

$$W = W_Q + \frac{1}{\mu} = \text{the average time spent in the system, including service}$$

$$L = \lambda W = \text{the average number of customers in the service system}$$

$$P_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!} P_0 & \text{for } n \le s \\ \dfrac{(\lambda/\mu)^n}{s!s^{n-s}} P_0 & \text{for } n > s \end{cases} = \text{the probability that } n \text{ customers are in the system at a given time}$$

Single Line Multi-server Queueing Equations

Entering the Weighbridge at the site entry

- Average time for a vehicle to get weighed on the weighbridge = 60s
- Number of vehicles that can be weighed at once = 1
- Inbound traffic flow is worst case 21 inbound trips in an hour. This is a worst case assuming every vehicle entering the site requires to be weighed. In reality some vehicles entering the site empty will already have a known empty weights and as such will not be required to stop at the weighbridge.

Based on multi-channel queuing theory, the resulting queueing results are shown below.

Multi-Server Queue Worksheet

Service Bays	1	Arrival Rate (vehicles/hour)	21	Wait Time in Each Bay (seconds)	60
Vehicles/Second IN	0.00583333	Vehicles/Second OUT (per bay)	0.01666667		
P0	0.65	rho	0.35	rho (single bay system assumed)	0.35

n	1st Term	Pn	P(>=n)
0	1	65.00%	35.00%
1	0.35	22.75%	12.25%
2	0.06125	7.96%	4.29%
3	0.00715	2.79%	1.50%
4	0.00063	0.98%	0.53%
5	4.4E-05	0.34%	0.18%
6	2.6E-06	0.12%	0.06%
7	1.3E-07	0.04%	0.02%
8	5.6E-09	0.01%	0.01%
9	2.2E-10	0.01%	0.00%
10	7.6E-12	0.00%	0.00%

Percentile	Number of Vehicles in System	Number of Vehicles Queued
50%	0	0
60%	0	0
70%	1	0
80%	1	0
85%	1	0
90%	2	1
95%	2	1
98%	3	2

The 98th percentile queue results in 2 vehicles queued which can be accommodated wholly on-site

Vehicles being loaded

- Average time for a vehicle to get get loaded with material = 180s
- Number of vehicles that can be loaded at once = 2

Inbound traffic flow is worst case 21 inbound trips in an hour. This is a worst case assuming every vehicle entering the site is empty and will be loaded with material. In reality some vehicles entering the site will unload and some will load.

Based on multi-channel queuing theory, the resulting queueing results are shown below.

Multi-Server Queue Worksheet

Service Bays	2	Arrival Rate (vehicles/hour)	21	Wait Time in Each Bay (seconds)	180
Vehicles/Second IN	0.00583333	Vehicles/Second OUT (per bay)	0.00555556		
P0	0.31147541	rho	1.05	rho (single bay system assumed)	0.525

n	1st Term	Pn	P(>=n)
0	1	31.15%	68.85%
1	1.05	32.70%	36.15%
2	0.55125	17.17%	18.98%
3	0.19294	9.01%	9.96%
4	0.05065	4.73%	5.23%
5	0.01064	2.48%	2.75%
6	0.00186	1.30%	1.44%
7	0.00028	0.68%	0.76%
8	3.7E-05	0.36%	0.40%
9	4.3E-06	0.19%	0.21%
10	4.5E-07	0.10%	0.11%

Percentile	Number of Vehicles in System	Number of Vehicles Queued
50%	1	0
60%	1	0
70%	2	0
80%	2	0
85%	3	1
90%	3	1
95%	5	3
98%	6	4

The 98th percentile queue results in 4 vehicles queued which can be accommodated wholly on-site.

It is reiterated that the above assessment is highly conservative as a portion of entering vehicles will be loaded and portion will be unloaded. This would allow for further parallelisation between loading and unloading trucks which would reduce the queue length. Nevertheless, the site can still accommodate a queue of 4 vehicles under this conservative assessment.